UCRL-TR-220863

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# Heat Loss by Helicity Injection II

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April 26, 2006

This work was performed under the auspices of the U.S. Department of Energy by University of California, Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

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April 14, 2006


## Abstract

Arguments are reviewed showing that helicity transport always flattens the temperature profile, yielding unit current amplification in SSPX and flat temperature profiles in RFP's whenever the dynamo is active. The argument is based on transport theory yielding a hyper-resistivity $\Lambda \approx (c^2/\omega_{pe}{}^2)\chi_e$ with electron thermal diffusivity $\chi_e$, valid for any process producing a random-walk in electron constants of motion in the unperturbed field. The theory could be tested by deriving $\Lambda$ from helicity transport in SSPX, by analogy with recent analysis yielding $\chi_e$ from heat transport. If the predicted ratio $\Lambda/\chi_e$ is confirmed, efforts to increase current amplification in SSPX must be based on scenarios consistent with slow helicity transport compared to heat transport (pulsed reactor, multipulse, neutral beam injection).


1. Introduction

The revival of interest in spheromaks that led to SSPX was motivated by high temperatures achieved in CTX [1]. This note is an update of Ref. [2], written in 1994, in which I tried to account for the high temperatures in CTX and, from this, to obtain a buildup scenario yielding high current amplification. It was noted that CTX temperatures could be explained by S-scaling [3] applied to the Rechester-Rosenbluth thermal diffusivity $\chi_{RR}$ in a tangled magnetic field [4], giving:

$$\partial(nT)/\partial t - \nabla\cdot n\chi_e\nabla T \quad = \quad \eta j^2 \quad = \quad (B^2/2\tau\mu_o) \qquad (1)$$

where we used $\mu_o\, j = \lambda B$ giving the ohmic decay time $\tau = (\mu_o\,/2\eta\lambda^2)$. For steady state with $\chi_e = \chi_{RR}$, we obtain:

$$\chi_{RR} \quad = \quad v_e L_C(\delta B^2/B^2) \qquad (2)$$

$$\beta \quad = \quad 2\mu_o nT/B^2 \quad = \quad (v_A/v_e)[1/(\delta B^2/B^2)S] \qquad (3)$$

In Eqs. (2) and (3), $\delta\mathbf{B}$ is the magnetic fluctuation relative to a mean field $\mathbf{B}$; $L_C \approx a$ is the correlation length with minor radius a; $v_e$ is the electron thermal speed; and $S = v_A\tau/a$ with Alfven speed $v_A$ .

For S scaling ($\delta B^2/B^2 = S^{-1}$), Eq. (3) gives $\beta = v_A/v_e \approx (m_e/m_i)^{1/3}$, which fit CTX results for two very different values of B and T. By similar arguments Connor and Taylor had predicted constant beta in RFP's [5]; and this prediction of constant $\beta \approx 5$ - 10 % seemed to be even better confirmed in SSPX. This seeming success in predicting temperatures led us to conclude, incorrectly, that even weak S-scaling fluctuations must transport helicity fast enough to exceed resistive losses, thereby building up the current. It was through NIMROD simulations that we came to realize that high temperatures were achieved only if helicity transport ceases. Then flux surfaces close, giving roughly ion classical transport that yields a similar $\beta \approx (m_e/m_i)^{1/4}$ [5], now thought to be the more likely explanation for high temperatures.

Here we show that helicity transport is always slower than heat transport. Then helicity transport across any width $\Delta$ should always flatten the temperature, giving the same result as if field lines connect across $\Delta$ , even if flux surfaces close intermittently.

In Section 2 and in Appendix A, we review helicity transport theory, giving flat temperature profiles in Section 3 which in turn accounts for low current amplification in Section 4. In Section 5 we suggest experimental tests, by extracting helicity transport coefficients from SSPX data much as the heat transport coefficient $\chi$ is extracted now. Section 6 discusses results in the context of buildup scenarios that might yet succeed in achieving high current amplification.

## 2. Helicity Transport Theory

Helicity transport is described by the hyper-resistive Ohm's Law, derived in Appendix A, and given by:

$$\begin{aligned} E_{||} &= (m/ne^2)\{\nu j_{||} - \nabla_\perp \cdot D_M \nabla_\perp j_{||}\} \\ &= \eta j_{||} - B^{-1}\nabla_\perp \cdot B^2 \Lambda \nabla_\perp \lambda \end{aligned} \tag{4}$$

$$\Lambda = (m/\mu_o ne^2) D_M = (c^2/\omega_{pe}^2) D_M \tag{5}$$

In the second line of Eq. (4) we have put results in helicity-conserving form [6] with $\lambda = \mu_o j_{||}/B$, though as yet we cannot justify moving n inside the derivative (see Appendix B).

Eq. (4) is the moment equation for electron momentum neglecting inertia, with collision frequency $\nu$ giving resistivity $\eta = (m/ne^2)\nu = \mu_o(c^2/\omega_{pe}{}^2)\nu$. Any kind of turbulence produces an additional effective collision frequency $\nabla_\perp \cdot D_M \nabla_\perp$ with momentum diffusion coefficient $D_M$.

According to Kaufman's transport theory in action-angle space discussed in Appendix A [10], any process that transports electron momentum by Eq. (4) makes a comparable contribution $\approx D_M$ to the electron heat diffusivity, giving:

$$\chi_e \geq D_M \tag{6}$$

and for magnetic perturbations, usually $\chi_e \leq \chi_{RR}$. This result, obtained by several authors, is more general than might appear from derivations of "kinetic" hyper-resistivity in papers such as Refs. [7, 8, 9]. For example, contrary to various discussions in the literature, Eqs. (5) and (6) apply specifically to Strauss's hyper-resistivity derived from the MHD dynamo $<\delta\mathbf{v}\mathrm{x}\delta\mathbf{B}>$ for tearing modes with $\chi_e \leq \chi_{RR}$ [11]. The correspondence of Kaufman's more general theory with MHD is discussed in Appendix B.

3. Flattening of the Temperature Profile

If instability is strong enough to cause helicity to be transported, the right hand side of Eq. (4) must be negative so that helicity flow exceeds resistive losses. By inspection, making the right hand side of Eq. (4) negative requires:

$$\chi_e \geq D_M > \nu\Delta^2 = (\omega_{pe}{}^2/\lambda^2c^2)(\Delta^2/\tau) \tag{7}$$

where we use Eq. (6), and $\nabla \approx 1/\Delta$. Introducing Eq. (7) into Eq. (1) for steady state and dividing by $n\chi_e T$ gives:

$$-\nabla\cdot n\chi_e \nabla T > n(\omega_{pe}{}^2/\lambda^2c^2)(\Delta^2/\tau)\nabla^2 T = (B^2/\tau\mu_o) \tag{8}$$

$$\delta T/T \equiv (\Delta^2 \nabla^2 T/T) < [1/\beta(\omega_{pe}{}^2/\lambda^2c^2)] \tag{9}$$

Here $\delta T$ is the temperature change across the scalelength $\Delta$ for local gradients in $j_{||}$ in Eq. (4). For typical parameters, $\delta T/T \ll 1$ even for $\beta \approx 1\%$. It is this that accounts for flat temperature profiles in MST whenever the dynamo is active, and for limitations on current amplification in

SSPX. Note that, for magnetic turbulence, this result does not depend explicitly on $\delta B/B$; instead, instability forces $\delta B/B$ to satisfy Eq. (7).

4. Flux and Current Amplification in Spheromaks

For the spheromak, Eq. (9) implies that during gun injection the average electron temperature T is equal to that in the flux core connected to the gun, namely, that for short open field lines [3]:

$$T \approx 0.4(V - V_S) \tag{10}$$

with gun voltage V and sheath voltage $V_S$. Thus, during buildup, we should use Eq. (10) in calculating the resistivity appearing in the Ohm's Law, Eq. (4).

To calculate helicity buildup, we dot Eq. (4) into **B** and integrate over any volume including the flux core:

$$dK/dt \equiv d/dt \int \mathbf{A}\cdot\mathbf{B} = 2V\Phi - \int d\mathbf{S}\cdot\boldsymbol{\Gamma} - K/\tau \tag{11}$$

$$\boldsymbol{\Gamma} = -2B^2\Lambda\nabla\lambda \tag{12}$$

where $\Phi$ is the gun bias flux and $\int d\mathbf{S}\cdot\boldsymbol{\Gamma}$ is helicity flow across the surface, usually taken zero if the volume includes the entire flux conserver. Dropping the surface term gives at any time during the buildup [12]:

$$K \approx \psi_{POL}\psi_{TOR} \approx \psi_{POL}{}^2 \leq 2V\Phi\tau \tag{13}$$

$$\psi_{POL}/\Phi \leq (2V\tau/\Phi)^{1/2} \tag{14}$$

Eq. (14) gives the flux amplification ($\psi_{POL}/\Phi$) with $\tau$ calculated for resistivity $\eta$ with T in Eq. (10). For large gun current [12], we can neglect the impedance that Eq. (11) presents to the gun and obtain the current amplification by substituting $V \geq I_{GUN} R_\Omega$ into Eq. (14), where $R_\Omega = \eta_C L B_C/\Phi$ is the resistance of the flux core with length L and area $\Phi/B_C$ where $B_C$ is the poloidal field at the geometric axis. Using $\psi_{POL} = 1/2\mu_o I_{TOR} a$ and $\tau = (\mu_o/2\eta\lambda^2)$ with $\lambda a \approx 2$, the result is [12]:

$$I_{TOR}/I_{GUN} \leq (B_C L/\mu_o I_{TOR}) \tag{15}$$

where, consistent with the flattened temperature profile, we took $\eta$ inside the separatrix to equal $\eta_C$ in the flux core. In the limit of large $I_{TOR}$, $B_C \propto I_{TOR}/a$ whereby the maximum current amplification $I_{TOR}/I_{GUN}$ approaches a constant $\propto L/a$ characteristic of the flux conserver, with a value of order unity in agreement with SSPX results [12].

Eqs. (14) and (15) can be derived from modified Taylor relaxation theory using minimization of the energy dissipation rather than minimizing the energy itself [13].

## 5. Experimental Tests of the Transport Theory

The most direct test of the theory would be obtained by substituting Eq. (6) into Eqs. (1) and (4) giving T and $\lambda$ profiles to be compared with experimental data. Lacking a reliable calculation of $\delta B$, one could simply take $D_M$ to be a constant in Eqs. (4) and (5), to be determined by fitting $\lambda$ profile data, as was done in Refs. [3] and [14] using the Ohm's Law of Eq. (4) to calculate the $\lambda$ profile using the Corsica code. The new feature would be a simultaneous T profile obtained by including Eq. (1) in Corsica in addition to the hyper-resistive Ohm's Law. One might include a second fitting parameter $\alpha$ in $\chi_e = \alpha D_M$, $\alpha$ being nominally unity according to the theory. One could also compare the empirically determined $D_M \approx \chi_{RR} \propto \delta B^2$ with that calculated from $\delta B$ in NIMROD simulations and edge probe measurements.

Indirect tests of the transport theory are provided by comparison of Eq. (10) with temperatures measured during helicity injection, and the flux and current amplification derived from Eq. (10) in Section 4.

Finally, NIMROD simulations have shown that spikes on the gun voltage represent events converting toroidal flux to poloidal flux, an almost-instantaneous helicity transport event [15]. Thus one might obtain direct evidence for maximum helicity transport rates and for temperature flattening during voltage spikes.

A voltage spike $\delta V$ is found by perturbing Eq. (11) applied to a volume including only the flux core with surface area $A_C$. If we ignore helicity buildup and loss inside the flux core, this gives:

$$2\Phi\delta V \approx \delta\int d\mathbf{S}\cdot\mathbf{\Gamma} = (2A_C B^2 \nabla\lambda)\delta\Lambda \qquad (16)$$

Here we neglected $\delta\nabla\lambda$ near the flux core. Dividing Eq. (16) by the steady state $2\langle V\rangle\Phi \approx (2A_C B^2\nabla\lambda)\langle\Lambda\rangle$ gives:

$$\delta\Lambda/\langle\Lambda\rangle \approx \delta V/\langle V\rangle \qquad (17)$$

Thus voltage spikes are indicative of the maximum $\Lambda$ near the flux core, as compared with the time-space average $\langle\Lambda\rangle$ obtained from Corsica fits.

A jump in $\Lambda$ during a voltage spike should produce a corresponding jump in $\chi_e$, most easily observed as a temperature flattening event inside a local region of intermittently closed flux where low turbulence allows T to grow beyond the limits of Eq. (10), as is observed in some NIMROD runs [15]. According to NIMROD, this usually occurs near the magnetic axis. Instantaneous measurements of temperature near the magnetic axis before and after a voltage spike, using the double-pulsed Thomson capability soon to be available, might be able to observe a high local temperature during a quiet time and subsequent collapse of the temperature during a spurt of helicity injection correlated with a voltage spike [16].

## 6. Conclusions

We have concluded from transport theory that helicity transport across any region flattens the electron temperature profile in that region, and it is this that limits current buildup in SSPX.

We have suggested ways that the transport theory could be tested. Indirect evidence based on limits on current amplification suggests that the theory is basically correct, and the discussion in Appendix A shows that the theory applies to any process that might transport helicity diffusively, not only tearing.

If further analysis continues to support the theory, efforts to increase current amplification in SSPX must be based on scenarios consistent with slow helicity transport. Three scenarios that meet this requirement are the pulsed reactor, multipulsing and current drive by neutral beams. There may be others.

The pulsed reactor could work because buildup is accomplished without current amplification [17]. The main issues are how much magnetic energy is lost in a transition from the Taylor state produced by electrostatic injection to a stable mode of decay, and whether a stable mode exists that is not supported by gun current at the edge. The latter point has gained support by Pearlstein's results showing equilibria with zero $\lambda$ at the edge that are stable to tearing in the straight-cylinder approximation [18].

Multipulsing, already explored to some extent [3], might have a better chance by reducing the bias flux and gun current after the initial formation phase [19], as is currently being explored on NIMROD [15]. Then helicity would be injected in a succession of small pulses. Again current amplification is not required for a single pulse. Success requires, first, that injection does not break flux surfaces in the interior, and secondly, that flux closure detaches each pulse from the gun, thereby allowing the newly injected helicity to merge into the spheromak already present [19]. Merger is aided by mutual attraction between toroidal current in a new pulse and that in the spheromak. Flux closure could be accomplished either inside the gun as the pulse leaves the gun, or in the flux conserver itself.

Thirdly, the possibility of a stable state with no gun current suggests that other methods of current drive such as neutral beams could build up and maintain this state, perhaps starting from a "target" produced by gun injection [18].

That helicity transport flattens $\nabla T$ has been suspected for many years [9], but the theory is complicated in detail and only now are experiments and NIMROD simulations sufficiently advanced to warrant the kind of experimental campaign needed to test the theory in detail.

**Acknowledgement**

I would like to thank Don Pearlstein for numerous and ongoing discussions of the topic of this paper, and Renato Gatto whose Ph.D. thesis is quoted in Ref. [21].

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## Appendix A. Hyper-Resistivity in Transport Theory

We derive transport following Mahajan, Hazeltine and Hitchcock, who applied Kaufman's quasi-linear theory [10] to Ohm's Law and heat transport in tokamaks [20]. Using Kaufman's approach, they were able to show why resistivity is approximately classical even when turbulence dominates heat transport, without recourse to details of the turbulence spectra. Here we use Kaufman's approach to compare heat transport and hyper-resistivity.

Kaufman's theory for axisymmetric tori recasts the Vlasov equation in phase space $\mathbf{x},\mathbf{v}$ as a continuity equation in action-angle variables $\mathbf{J},\boldsymbol{\theta}$ where the $\mathbf{J}$ components are adiabatic approximations to canonical momenta for the mean field [10]:

$$\partial f/\partial t \;+\; \partial/\partial\boldsymbol{\theta}\cdot[(d\boldsymbol{\theta}/dt)f] \;+\; \partial/\partial\mathbf{J}\cdot[(d\mathbf{J}/dt)f] \;=\; \text{collisions} \qquad \text{(A1)}$$

where $\partial/\partial t$ is taken at constant $\mathbf{J},\boldsymbol{\theta}$, and so on. In equilibrium, the dynamics is Hamiltonian so that $d\boldsymbol{\theta}/dt$ and $d\mathbf{J}/dt$ can be removed outside the derivatives. Correspondence with the usual Vlasov equation is found by transforming back to $\mathbf{x},\mathbf{v}$, giving, for $\mathbf{J} \approx \mathbf{p}_o = m\mathbf{v} + q\mathbf{A}_o$ with equilibrium vector potential $\mathbf{A}_o$:

$$(\partial f/\partial t)_{\mathbf{J},\boldsymbol{\theta}} \approx (\partial f/\partial t)_{\mathbf{p},\mathbf{x}} = \partial f/\partial t + (d\mathbf{x}/dt)_{\mathbf{J},\boldsymbol{\theta}}\cdot(\partial f/\partial\mathbf{x}) - q(\partial\mathbf{A}_o/\partial t)\cdot(\partial f/\partial\mathbf{v}) \qquad \text{(A2)}$$

$$\begin{aligned} d\mathbf{J}/dt \approx d\mathbf{p}_o/dt &= m(d\mathbf{v}/dt) + q[\partial\mathbf{A}_o/\partial t + (\mathbf{v}\cdot\nabla)\mathbf{A}_o] \\ &= q[\mathbf{E} + \mathbf{v}\mathrm{x}\mathbf{B}] + q[\partial\mathbf{A}_o/\partial t + (\mathbf{v}\cdot\nabla)\mathbf{A}_o] \qquad \text{(A3)} \end{aligned}$$

To obtain Kaufman's quasi-linear equation, we linearize Eq. (A1) and substitute the solution into Eq. (A1) averaged over $\boldsymbol{\theta}$ (with notation $<\cdot\cdot> = \int d\boldsymbol{\theta}/(2\pi)^3$), $<f> = f_o$). Neglecting collisions, the result is:

$$\partial f_o/\partial t = \partial/\partial\mathbf{J}\cdot\underline{\mathbf{D}}\cdot\partial f_o/\partial\mathbf{J} \qquad \text{(A4)}$$

$$\partial f_1/\partial t + (d\boldsymbol{\theta}/dt)_o\cdot\partial f_1/\partial\boldsymbol{\theta} \equiv Lf_1 = -\,\partial/\partial\mathbf{J}\cdot[(d\mathbf{J}/dt)_1 f_o] \qquad \text{(A5)}$$

$$\underline{\mathbf{D}} = <(d\mathbf{J}/dt)_1 L^{-1}(d\mathbf{J}/dt)_1> \qquad \text{(A6)}$$

The generality of Kaufman's approach lies in the fact that, while he conveniently approximates $\mathbf{p}_o \approx \mathbf{J}$ in equilibrium, the form of the transport equation remains the same for $(d\mathbf{J}/dt)_1$ evaluated for any exact turbulence spectrum for any $\omega$, $\mathbf{k}$.

Finally, we obtain Ohm's Law by taking the moment of Eq. (4) for electron momentum, giving, with electron mass m and q = - e:

$$q\int d\mathbf{v}\; m\mathbf{v}_{||}\,(\partial f_o/\partial t)_{\mathbf{J},\boldsymbol{\theta}} \quad = \quad q\int d\mathbf{v}\; m\mathbf{v}_{||}\;(\partial/\partial\mathbf{J}\cdot\underline{\mathbf{D}}\cdot\partial f_o/\partial\mathbf{J}) \qquad \text{(A7)}$$

On the left, we substitute the average of the right hand side of Eq. (A2) expressed in $\mathbf{x}$,$\mathbf{v}$ variables. Integrating by parts over $\mathbf{v}$ gives $qn_o\mathbf{E}_o$ with inductive mean field $\mathbf{E}_o = - \partial\mathbf{A}_o/\partial t$, plus two terms that we will drop hereafter, namely, inertia from $\partial f_o/\partial t$ and the Ware pinch from $(d\mathbf{x}/dt)_{\mathbf{J},\boldsymbol{\theta}}$ [20]. The electrostatic field drops out in averaging over $\boldsymbol{\theta}$ (no contribution to the loop voltage.) On the right, following Refs. [10] and [20], we replace $\int d\mathbf{v}$ at fixed $\mathbf{x}$ by $\int d\mathbf{J}d\boldsymbol{\theta}\;\delta(\mathbf{x} - \mathbf{r}(\mathbf{J},\boldsymbol{\theta},t))$ where $\mathbf{r}(\mathbf{J},\boldsymbol{\theta},t)$ is the electron orbit, whereby the delta function adds up contributions to the current from all electrons passing through $\mathbf{x}$. Then integrating by parts on $\mathbf{J}$ gives two terms, one being radial diffusion and the other, coming from $\partial\mathbf{v}_{||}/\partial\mathbf{J}$, being a "source" term giving rise to both resistance and bootstrap current for collisions. This source term is proportional to $k_{||}$ and hence usually negligible for collective turbulence [20, 21].

With these understandings, and restoring the collision frequency $\nu$, Eq. (A7) yields:

$$E_{o||} \quad = \quad (m/ne^2)\{\nu j_{||} \quad - \quad \nabla_\perp\cdot D_M\nabla_\perp\, j_{||}\} \qquad \text{(A8)}$$

This is our main result, giving the Ohm's Law of Eq. (4), with a momentum diffusion coefficient $D_M$ that would also appear in the energy moment of Eq. (A4), giving Eq. (6) of the main text. The characteristically small ratio $\Lambda/D_M = (c^2/\omega_{pe}{}^2)$ yielding the main results of this paper arises from the factor $(m/ne^2)$, which accounts for inductive energy as it does in the resistive term above with collision frequency $\nu$. Turbulent transport merely contributes an additive "collision frequency" $\approx \nabla_\perp\cdot D_M\nabla_\perp$.

Following Mynick [22], Gatto has derived Ohm's Law and heat transport from a Balescu-Lenard extension of Kaufman's theory, giving [21]:

$$\partial f_o/\partial t \quad = \quad - \partial/\partial\mathbf{J}\cdot[\,\underline{\mathbf{D}}\cdot\partial f_o/\partial\mathbf{J} - \mathbf{F}f] \qquad \text{(A9)}$$

where the vector **F** is a friction term, analogous to the "drag" term in the Fokker-Planck equation in velocity space, with essentially the same characteristic frequency $\nabla_\perp \cdot D_M \nabla_\perp$. Gatto showed that **F** does not change the form of Eq. (4) [21].

Appendix B. Correspondence with MHD, Other Theory

As noted throughout this paper, the virtue of Kaufman's formulation is its generality, adding confidence in our conclusion that hyper-resistive transport of helicity is always slow compared to heat transport, for any kind of turbulence. Since Kaufman's theory includes MHD as a limit, we would expect MHD hyper-resistivity to satisfy Eqs. (4) - (6) and indeed this is the case for Strauss's hyper-resistivity for tearing modes, giving [11]:

$$\langle \delta \mathbf{v} \times \delta \mathbf{B} \rangle = - B^{-1} \nabla_\perp \cdot B^2 \, \Lambda_{MHD} \, \nabla_\perp \lambda \qquad \text{(B1)}$$

$$\Lambda_{MHD} = [\gamma/(\partial k_{||}/\partial r)^2](\delta B^2/B^2) = F(c^2/\omega_{pe}^2)\chi_{RR} \qquad \text{(B2)}$$

$$F = [(\Delta' a)/\lambda_{MFP} L_C](a \partial k_{||}/\partial r)^2 = (\Delta' a)(L_C/\lambda_{MFP})(L_q/nL_C)^2 \qquad \text{(B3)}$$

To compare with Eqs. (5) and (6) in the main text, we have rewritten $\Lambda_{MHD}$ in the form of Eq. (5) with a dimensionless multiplier F obtained as follows. The first expression on the right side of Eq. (B3) is the result of Ref. [11] (see Eq. (10) of that paper). We take the non-linear growth rate $\gamma$, as was done in Ref. [11]. However, rather than Strauss's "direct interaction approximation," we use the more conventional non-linear growth rate in the Rutherford regime, giving $\gamma = d(w/a)/dt = (\eta/a^2\mu_o)(\Delta' a)$ for the resistive growth of island widths w. For this case, it is $\eta$ in the Rutherford $\gamma$ that introduces the ubiquitous factor $c^2/\omega_{pe}^2$ . On the far right in Eq. (B2), we replace $(\eta/\mu_o) = (c^2/\omega_{pe}^2)\nu$ with electron-ion collision frequency $\nu$ giving $\lambda_{MFP} = v_e/\nu$ in Eq. (B3). Also in Eq. (B3), we take $(a\partial k_{||}/\partial r) \approx n/L_q$ where n is the toroidal mode number and $L_q = q/(dq/dr)$ is the shear length. In writing $\chi_{RR} = v_e L_C\, (\delta B^2/B^2)$ in Eq. (B6), we assume $L_C > \lambda_{MFP}$ giving a small factor $(L_C/\lambda_{MFP})$ in F; if instead $L_C < \lambda_{MFP}$, $L_C$ is replaced by $\lambda_{MFP}$ in $\chi_{RR}$, in which $\lambda_{MFP}$ cancels. Also, usually $(\Delta' a) \leq 1$. With these understandings, we see that $F \leq 1$ for most purposes and in any case $F << (a^2\omega_{pe}^2/c^2)$, giving $\Lambda_{MHD}$ similar to Eq. (5) with $\chi_e \leq \chi_{RR}$.

An important feature of MHD hyper-resistivity is the fact that it conserves helicity. This follows from the general form for Ohm's Law [6]:

$$\mathbf{E} + \mathbf{v}\mathrm{x}\mathbf{B} \quad = \quad \mathbf{R} \tag{B4}$$

where **v** is the electron fluid velocity (by including the Hall term) and **R** includes inertia, collisions and turbulent resistivity, viscosity, hyper-resistivity etc. Then the helicity K satisfies:

$$dK/dt \quad = \quad -2\int d\mathbf{x}\ \mathbf{E}\cdot\mathbf{B} \quad = \quad -2\int d\mathbf{x}\ \mathbf{R}\cdot\mathbf{B} \tag{B5}$$

Helicity is conserved if **R** is small, as assumed in reduced MHD theory, and it is better conserved than is the magnetic energy with finite resistivity [6].

The MHD hyper-resistivity derived from **v**x**B** has the form of the second line of Eq. (4), which is manifestly helicity conserving [6, 11, 23]. In reduced MHD used in Ref. [11], this follows directly from the collisionless Ohm's Law [11]:

$$E_{oz} \quad = -\,\partial/\partial t\ \delta A_z \quad = \quad -<\delta\mathbf{v}\mathrm{x}\delta\mathbf{B}>_z \quad = \quad \nabla\cdot<\delta\mathbf{v}\delta A_z> \tag{B6}$$

$$\mathbf{R}\cdot\mathbf{B} \quad = -<\delta\mathbf{v}\mathrm{x}\delta\mathbf{B}>_z B_z \quad \approx \quad -\nabla\cdot(B_z <\delta\mathbf{v}\delta A_z>) \tag{B7}$$

The last step in Eq. (B6) follows in reduced MHD from $\mathbf{B} = \nabla\delta A_z \mathrm{x}\mathbf{z}$ with **z** along the mean field direction, giving $\delta\mathbf{v}\mathrm{x}\delta\mathbf{B}$ as the convective derivative of $A_z\mathbf{z}$ ($\delta\mathbf{v}\cdot\nabla\delta A_z = \nabla\cdot(\delta\mathbf{v}\delta A_z)$ for $\nabla\cdot\delta\mathbf{v} = 0$; though in fact MHD conservation of helicity does not depend on incompressible flow [25]). In Eq. (B7), we passed the relatively smooth mean field $B_z$ through the derivative. Then ($-B_z <\delta\mathbf{v}\delta A_z>$) is half the helicity flux.

The Ohm's Law derived in this paper is not manifestly helicity conserving; in particular, n stands outside the divergence in Eqs. (4) and (A8). Even if we ignore gradients in the relatively smooth mean field, transforming to the helicity-conserving form in the second line of Eq. (4) would add a correction $\propto \partial(\ln n)/\partial r$.

A contribution to **R** from $\mathbf{v}\cdot\nabla f$, omitted in reduced MHD, is also not obviously helicity conserving. Properly evaluated, this term makes a contribution comparable to the **v**x**B** contribution, as can be seen from estimating $\nabla\cdot\int d\mathbf{v}\ m v_{||}\ (v_{1r}\ f_1)$ with $f_1 \approx (L_C/v_e)\ v_{1r}\nabla f_o$ (in the limit $\omega \rightarrow 0$ [21, 24]) and $v_{1r} \approx v_{||}\ (\delta B/B)$. Kaufman's approach takes account of both $\mathbf{v}\cdot\nabla f$ and the <**v**x**B**> dynamo simultaneously [see Eq. (A3), containing both **v**x**B** explicitly and the convective derivative of **A**].

Nonetheless, the deeper reasons for the better conservation of K than energy apply both to turbulence and to collisions [6]. For any component $\mathbf{B}_i$ of the field expanded in eigenstates of curl **B** with energy $E_i$, the helicity $K_i = E_i / \lambda_i$ so that, in summing over states with $\lambda_i$ 's of varying sign, most of the helicity resides in the slowest-decaying relaxed state with lowest $\lambda$ (the Taylor state).

Two other points concerning Gatto's Ohm's Law [21]. First, an earlier debate whether Rechester-Rosenbluth heat and momentum transport are reduced by ambipolarity [24] appears to have been resolved in favor of transport at the electron speed as assumed here [26, 27]. Secondly, the profile consistency conjecture of Ref. [28] explored by Gatto yields $\Lambda/\chi_e$ larger than our general result by a factor $v_e^2/\underline{v}^2$ with $\underline{v}$ = j/ne.